Todos os exercícios das páginas 638 e 657-658 do livro do Reginaldo: Introdução às equações diferenciais ordinárias

1-11 Use séries de potências para resolver a equação diferencial.

1. $y' - y = 0$

2. $y' = xy$

3. $y' = x^2 y$

4. $(x - 3)y' + 2y = 0$

5. $y'' + xy' + y = 0$

6. $y'' = y$

7. $(x - 1)y'' + y' = 0$

8. $y'' = xy$

9. $y' - xy' - y = 0, \quad y(0) = 1, \quad y'(0) = 0$

10. $y'' + x^2 y = 0, \quad y(0) = 1, \quad y'(0) = 0$

1. $c_0 \sum_{n=0}^{\infty} \frac{x^n}{n!} = c_0 e^x$

3. $c_0 \sum_{n=0}^{\infty} \frac{x^{3n}}{3^n n!} = c_0 e^{x^3/3}$

5. $c_0 \sum_{n=0}^{\infty} \frac{(-1)^n}{2^n n!} x^{2n} + c_1 \sum_{n=0}^{\infty} \frac{(-2)^n n!}{(2n+1)!} x^{2n+1}$

7. $c_0 + c_1 \sum_{n=1}^{\infty} \frac{x^n}{n} = c_0 - c_1 \ln(1 - x)$ for $|x| < 1$

9. $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^n n!} = e^{x^2/2}$

11. $x + \sum_{n=0}^{\infty} \frac{(-1)^n 2^2 5^2 \cdot \cdots \cdot (3n-1)^2}{(3n+1)} x^{3n+1}$

Calcule a solução em série centrada no ponto ordinário $x = 0$ de cada uma das EDOs abaixo:

(a) $y'' = xy$ **(b)** $y'' - 2xy' + y = 0$ **(c)** $y'' + x^2y' + xy = 0$ **(d)** $(x^2 + 2)y'' + 3xy' - y = 0$

<u>Respostas</u>:

(a) $y(x) = a_0\left(1 + \frac{1}{2\cdot3}x^3 + \frac{1}{2\cdot3\cdot5\cdot6}x^6 + \frac{1}{2\cdot3\cdot5\cdot6\cdot8\cdot9}x^9 + \cdot\right) + a_1\left(x + \frac{1}{3\cdot4}x^4 + \frac{1}{3\cdot4\cdot6\cdot7}x^7 + \frac{1}{3\cdot4\cdot6\cdot7\cdot9\cdot10}x^{10} + \cdots\right)$

(b) $y(x) = a_0\left(1 - \frac{1}{2!}x^2 - \frac{3}{4!}x^4 - \frac{21}{6!}x^6 - \cdots\right) + a_1\left(x + \frac{1}{3!}x^3 - \frac{5}{5!}x^5 + \frac{45}{7!}x^7 + \cdots\right)$

(c) $y(x) = a_0\left(1 - \frac{1}{3!}x^3 + \frac{4^2}{6!}x^6 - \frac{4^2\,7^2}{9!}x^9 + \cdots\right) + a_1\left(x - \frac{2^2}{4!}x^4 + \frac{2^2\,5^2}{7!}x^7 - \frac{2^2\,5^2\,8^2}{10!}x^{10} + \cdots\right)$

(d) $y(x) = a_0\left(1 + \frac{1}{4}x^2 - \frac{7}{4!\,4}x^4 + \frac{7\cdot23}{6!\,8}x^6 - \cdots\right) + a_1\left(x - \frac{1}{3!}x^3 + \frac{14}{5!\,2}x^5 - \frac{14\cdot34}{7!\,4}x^7 - \cdots\right)$